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Modern Logistics & Supply Chain Management

ML & SCM

Transportation Models

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Winter 2016

As gold which he cannot spend
will make no man rich,
so knowledge which he cannot apply
will make no man wise.
Samuel Johnson: The Idler No. 84

Contents & Learning Objectives

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- The Transportation Model
- The Transshipment Model
- The Assignment Model

USA:50% unutilised
Loss of 31 billion
Schneider.com

90% of all coal
67% of cars
68% of paper

90% of total world trade
7.7 billion tonnes [2008]
32 trillion tonne-miles

1% of tonnage
FedEx, UPS, DHL

The Transportation Model: Characteristics

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- A product is transported from a number of sources to a number of destinations at the ***minimum possible cost***.
- Each ***source is able to supply a fixed number*** of units of the product, and ***each destination has a fixed demand*** for the product.
- The linear programming model has ***constraints for supply*** at each source ***and demand*** at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.

Transportation Model Example
Problem Definition and Data

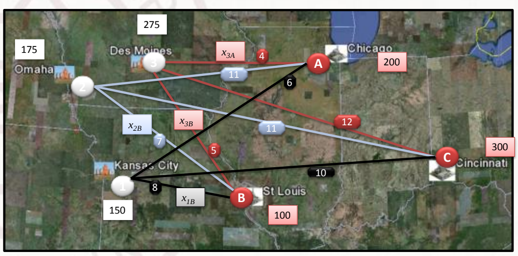
How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation?

Grain Elevator	Supply	Mill	Demand
1. Kansas City	150	A. Chicago	200
2. Omaha	175	B. St. Louis	100
3. Des Moines	275	C. Cincinnati	300
Total	600 tons	Total	600 tons

Transport Cost from Grain Elevator to Mill ($/ton)

Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$ 6	$ 8	$ 10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Transportation Network



Transportation Model Example
Model Formulation

Minimize $Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$

subject to:

$x_{1A} + x_{1B} + x_{1C} = 150$
$x_{2A} + x_{2B} + x_{2C} = 175$
$x_{3A} + x_{3B} + x_{3C} = 275$

supply constraints (from/source)

$x_{1A} + x_{2A} + x_{3A} = 200$
$x_{1B} + x_{2B} + x_{3B} = 100$
$x_{1C} + x_{2C} + x_{3C} = 300$

demand constraints (to/destination)

$x_{ij} \geq 0$

non-negativity (nothing can be returned)

Is this problem balanced?

Yes, because demand is equal to supply.

x_{ij} = tons of wheat from each grain elevator i ∈ {1, 2, 3}, to each mill j ∈ { A, B, C }

Transportation Model Example Model Formulation

$$D = (d_{ij}) = \begin{pmatrix} 6 & 8 & 10 \\ 7 & 11 & 11 \\ 4 & 5 & 12 \end{pmatrix} \in \mathbb{R}^{n \times m}$$

$$X = (x_{ij})$$

$$f(X) = \sum_{i=1}^{3} \sum_{j \in \{A,B,C\}} d_{ij} x_{ij} \qquad f(X) = \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij}$$

$$b_i^s = \begin{pmatrix} 150 \\ 175 \\ 275 \end{pmatrix} \qquad \sum_{j=1}^{m} x_{ij} = b_i^s \;\; \forall i \in \{1, \cdots, n\}$$

$$b_j^d = \begin{pmatrix} 200 \\ 100 \\ 300 \end{pmatrix} \qquad \sum_{i=1}^{n} x_{ij} = b_j^d \;\; \forall j \in \{1, \cdots, m\}$$

A feasible solution

	to	Chicago	St. Louis	Cincinnati	
from		A	B	C	supply
Kansas City	1	150	0	0	150
Omaha	2	50	100	25	175
Des Moines	3	0	0	275	275
	demand	~~200~~ 50	100	300	

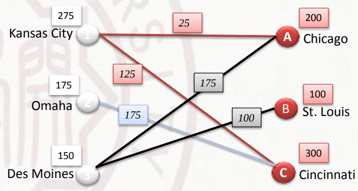
- Only constraints needed
  - No costs required
- Northwest corner rule
  - Start at top left and work your way down

Transportation methods

- Transportation Simplex Method
  - Initialisation – construct an initial basic feasible (BF) solution
    - Northwest corner rule
    - Vogel's approximation method
    - Russell's approximation method
  - Optimality test – if $c_{ij} - u_i - v_j \geq 0$ for every (i,j)
    - u_i row difference, v_j column difference (smallest minus next-to-smallest)
  - Iteration
    - Determine entering basic variable
    - Determine leaving basic variable
    - Determine new BF solution
- Find details in…
  - Hillier Introduction to Operations Research, chapter 8.2 (p319ff)

Optimal solution

Transportation Network Solution

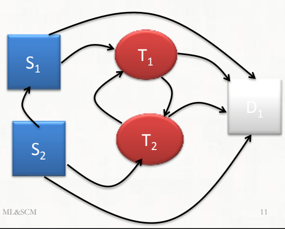


	A	B	C	
1	25	0	125	150
2	0	0	175	175
3	175	100	0	275
	200	100	300	

The solution value for the optimal solution is $4,525per month.

The Transshipment Model Characteristics

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations.
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these

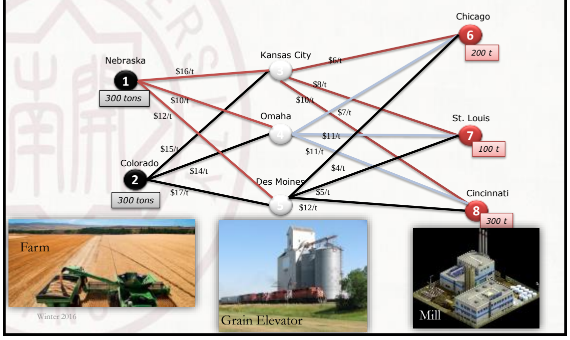


Transshipment Model Example Problem Definition and Data

Extension of the transportation model in which intermediate transshipment points are added between sources and destinations.

		1	2	3
		Kansas City	Omaha	Des Moines
1	Nebraska	$ 16	$ 10	$ 12
2	Coloardo	$ 15	$ 14	$ 17

Transshipment Model
Network routes



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Transshipment Model Example
Model Formulation

Minimize $Z = \$16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} + 17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} + 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}$

Sum of total costs = sum of: cost and quantity products

subject to:

$$x_{13} + x_{14} + x_{15} = 300$$
$$x_{23} + x_{24} + x_{25} = 300$$

Source constraints = available supply

$$x_{36} + x_{46} + x_{56} = 200$$
$$x_{37} + x_{47} + x_{57} = 100$$
$$x_{38} + x_{48} + x_{58} = 300$$

Destination constraints = required demand

$$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$$
$$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$$
$$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$$

Transhipment (no goods are left in these nodes)

$$x_{ij} \geq 0$$

Quantities

Winter 2016 ML&SCM 14

The Assignment Model
Characteristics

- Special form of linear programming model similar to the transportation model.
- ***Supply*** at each source ***and demand*** at each destination ***limited to one unit***.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.

Winter 2016 ML&SCM 15

Assignment Model Example
Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

	Game Sites			
Officials	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Assignment Model Example
Model Formulation

Minimize $Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} + 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} + 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC}$

subject to:

$x_{AR} + x_{AA} + x_{AD} + x_{AC} = 1$
$x_{BR} + x_{BA} + x_{BD} + x_{BC} = 1$
$x_{CR} + x_{CA} + x_{CD} + x_{CC} = 1$
$x_{DR} + x_{DA} + x_{DD} + x_{DC} = 1$

Supply constraints

$x_{AR} + x_{BR} + x_{CR} + x_{DR} = 1$
$x_{AA} + x_{BA} + x_{CA} + x_{DA} = 1$
$x_{AD} + x_{BD} + x_{CD} + x_{DD} = 1$
$x_{AC} + x_{BC} + x_{CC} + x_{DC} = 1$

Demand constraints

$x_{ij} \geq 0$

Assignment Model Example
Assignment Network Solution

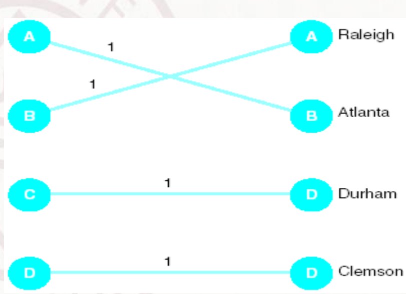


Figure 6.5

Applications
National grids

- National grids
  - Electricity
  - Gas
  - Water
  - "Telecommunication"

New gas supply projects

SDH/SONET european network

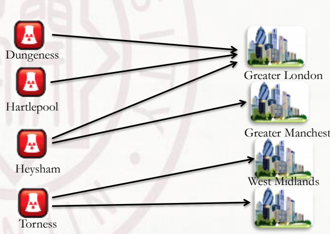
Water supply

Application
Nuclear power in the UK

- Provides about 20% of UK's required energy
- Supplies electrical output
  - Dungeness B: 1040 MW
  - Hartlepool: 1180 MW
  - …
    - see EDF Energy
- Demand (mainly in built up areas)
  - Greater London: 9.78M
  - Greater Manchester: 2.55M
  - West Midlands: 2.44M
  - ….
  - Assumption: electrical demand proportional to population
- Assume "transportation" cost depend on distance

Application
Nuclear power in the UK

- How should the energy be provided to the built-up areas in the most cost efficient way?



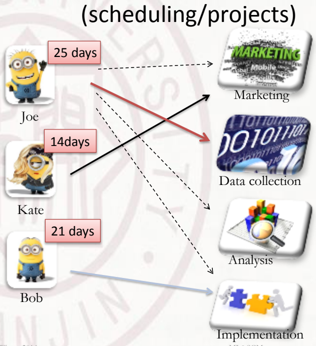
Other Applications

- Distribution networks
- Supply chains
  - Transshipment
- Scheduling
- Project Management

Assignment (scheduling/projects)

- Supply = resources (people)
  - Available time
- Demand = tasks
  - Time required for task
- Person to task
  - "Skills"
    - Critique: someone not skilled might take longer for task and quality might not be the same
      - Counter argument: the model attempts to prevent this

Assignment (scheduling/projects)



Skills	Marketing	Data Collection	Analysis	Implementation	supply
Joe	20%	90%	80%	20%	25 d
Kate	100%	10%	20%	90%	14 d
Bob	50%	50%	50%	60%	21 d
demand	10 d	15 d	12 d	20 d	57d\60d

Solution					
Joe	-	15.0	10.0	-	25 d
Kate	10.0	-	-	4.0	14 d
Bob	-	-	2.0	16.0	18 d
	10 d	15 d	12 d	20 d	57d\57d

Solution value
45.7

- we maximised
- uncertain data was used

Recap – "the end is near"

- What is a transportation model?
  – Demand is fulfilled by suppliers using links such that transportation costs are minimised.
- What is a transshipment model?
  – A generalisation of the transportation model by allowing intermediate "nodes".
- What is an assignment model?
  – A specialisation of the transportation model, where supply and demand is one (usually).
- Do you know any applications?
  – Supply chain, national grid, distribution networks

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The End

- Any questions?

Winter 2016 ML&SCM 26

Appendix

- References
- Transportation with Excel
- Transshipment with Excel
- Assignment with Excel
- Another example

Winter 2016 ML&SCM 27

References/Literature

- Introduction to management science: a modeling and case studies approach with spreadsheets - Hillier, Frederick S., Hillier, Mark S., Schmedders, Karl, Stephens, Molly
  - Chapter: 3 (p64-123)
- Introduction to operations research - Hillier, Frederick S., Lieberman, Gerald J. 2010
  - Chapter: 8 (p304-357)
- Network flows: theory, algorithms, and applications - Ahuja, Ravindra K., Magnanti, Thomas L., Orlin, James B. c1993
  - Chapter: 12 (p461
- Issues in operations management: [a management science approach] - Taylor, Bernard W., Garn, Wolfgang 2011
- Integer and combinatorial optimization - Nemhauser, George L., Wolsey, Laurence A. c1988

Winter 2016 ML&SCM 28

General Notes

- Part of a class of LP problems known as *network flow* models.
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure).
  - Detailed descriptions of methods are contained on Taylor's website (http://wps.prenhall.com/bp_taylor_introms_10/128/32822/8402671.cw/content/index.html)
- Text focuses on ***model formulation*** and solution with Excel and QM for windows.

Winter 2016 ML&SCM 29

Transportation Model Example
Transportation Network Routes

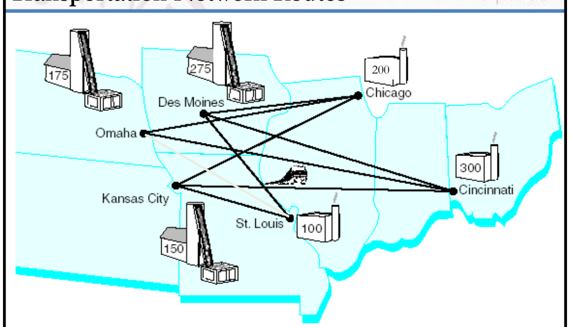
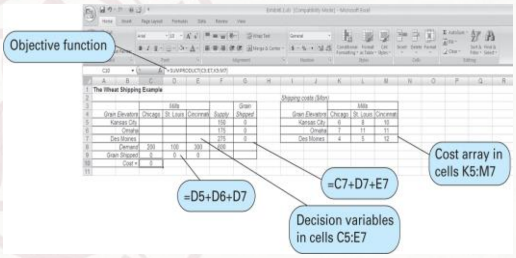


Figure 6.1 Network of Transportation Routes for Wheat Shipments

Winter 2016 ML&SCM 30

Transportation Model Example
Computer Solution with Excel (1 of 4)



Winter 2016 ML&SCM 31

Transportation Model Example
Computer Solution with Excel (2 of 4)

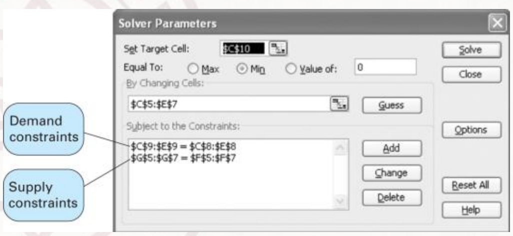


Exhibit 6.2

Winter 2016 ML&SCM 32

Transportation Model Example
Computer Solution with Excel (3 of 4)

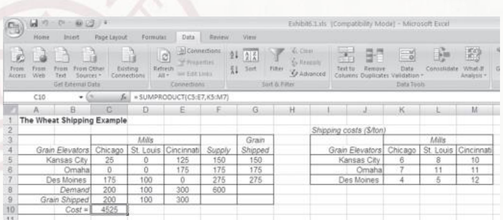


Exhibit 6.3

Winter 2016 ML&SCM 33

Transshipment Model Example
Transshipment Network Routes

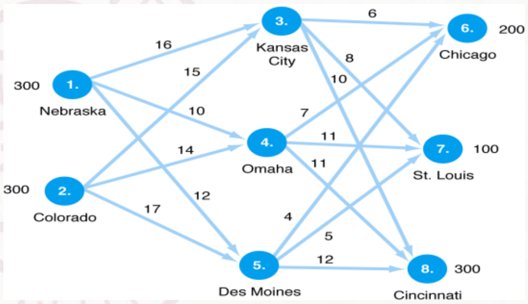


Figure 6.3 Network of Transshipment Routes

Transshipment Model Example
Computer Solution with Excel (1 of 3)

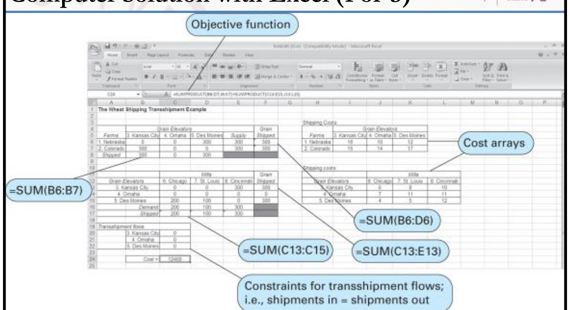


Exhibit 6.10

Transshipment Model Example
Computer Solution with Excel (2 of 3)

Exhibit 6.11

Transshipment Model Example
Network Solution for Wheat Shipping (3 of 3)

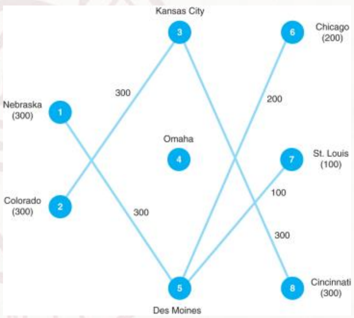


Figure 6.4

Assignment Model Example
Computer Solution with Excel (1 of 3)

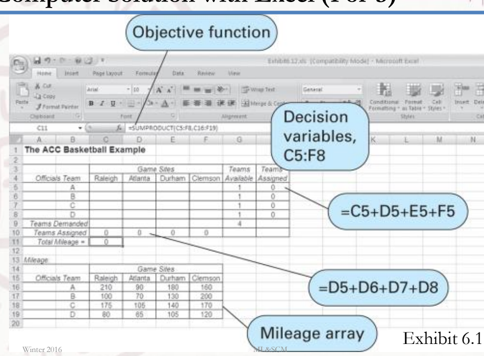


Exhibit 6.12

Assignment Model Example
Computer Solution with Excel (2 of 3)

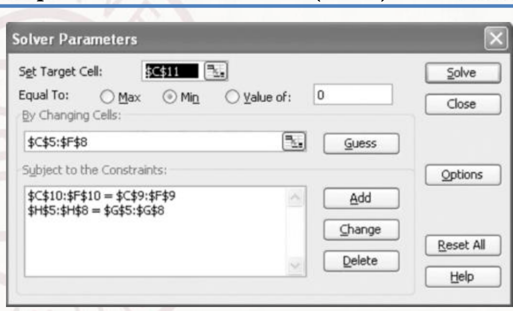
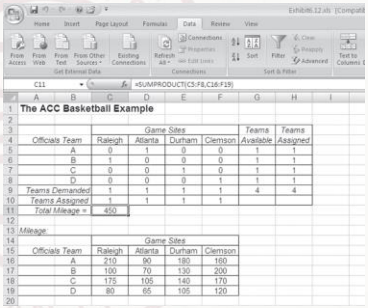


Exhibit 6.13

Assignment Model Example
Computer Solution with Excel (3 of 3)



C11 =SUMPRODUCT(C5:F8,C16:F19)

The ACC Basketball Example

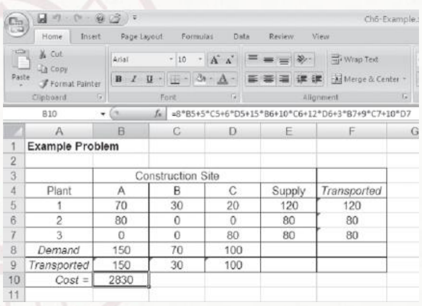
	Game Sites				Teams	Teams
Officials Team	Raleigh	Atlanta	Durham	Clemson	Available	Assigned
A	0	1	0	0	1	1
B	1	0	0	0	1	1
C	0	0	1	0	1	1
D	0	0	0	1	1	1
Teams Demanded	1	1	1	1	4	4
Teams Assigned	1	1	1	1		
Total Mileage =	450					

Mileage:

	Game Sites			
Officials Team	Raleigh	Atlanta	Durham	Clemson
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Exhibit 6.14

Example Problem Solution
Computer Solution with Excel



B10 =8*B5+5*C5+6*D5+15*B6+10*C6+12*D6+3*B7+9*C7+10*D7

Example Problem

	Construction Site				
Plant	A	B	C	Supply	*Transported*
1	70	30	20	120	120
2	80	0	0	80	80
3	0	0	80	80	80
Demand	150	70	100		
Transported	150	30	100		
Cost =	2830				

Example Problem Solution
Transportation Problem Statement

Determine the linear programming model formulation and solve using Excel:

	Construction site			
Plant	**A**	**B**	**C**	**Supply (tons)**
1	**$ 8**	**$ 5**	**$ 6**	**120**
2	**15**	**10**	**12**	**80**
3	**3**	**9**	**10**	**80**
Demand (tons)	**150**	**70**	**100**	

Example Problem Solution
Model Formulation

Minimize $Z = \$8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} + 3x_{3A} + 9x_{3B} + 10x_{3C}$

subject to:

$$x_{1A} + x_{1B} + x_{1C} = 120$$
$$x_{2A} + x_{2B} + x_{2C} = 80$$
$$x_{3A} + x_{3B} + x_{3C} = 80$$
$$x_{1A} + x_{2A} + x_{3A} \le 150$$
$$x_{1B} + x_{2B} + x_{3B} \le 70$$
$$x_{1C} + x_{2C} + x_{3C} \le 100$$
$$x_{ij} \ge 0$$

Winter 2016 ML&SCM 43